Technical Notes

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Material Coupling in the Analysis of Orthotropic Layered Shells

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IN the analysis of shells that are orthotropic and that may consist of more than one layer, it is important to consider carefully certain effects of material properties. Some of these effects are illustrated herein by several specific examples.

The governing equations for the analysis of asymmetrically loaded orthotropic shells of revolution have been reduced to a coupled set of four second-order differential equations for each Fourier harmonic by Tene.[1] A numerical analysis for static problems is performed here in the manner described in Ref. 2 with, however, a parabolic scheme to approximate derivatives at the boundaries as suggested in Ref. 3. For dynamic analysis, stepwise time integration is carried out using the unconditionally stable Houbolt technique, which has been discussed in Ref. 4, and used successfully for isotropic cylindrical shells. Of particular interest here are the stress resultant-strain relations [Eqs. (2), Ref. 1] and the integrals defining the coefficients [Eqs. (3), Ref. 1]. The notation of Ref. 1 has been retained.

The following example was chosen to illustrate the effects of coupling between extensional and bending behavior. For cross sections having material properties symmetric about the reference surface, coupling is eliminated. These uncoupled equations [Eqs. (2) of Ref. 1 with $C_2^{(1)} = C_{12}^{(1)} = G^{(1)} = 0$] have often been applied to problems in which certain coupling coefficients were actually not zero. Here, the solution is obtained for a two-layered orthotropic cylinder with constant properties in each layer subjected to a normal external pressure of the form $q = -\cos n\theta$. The geometric and material parameters, representative of practical configurations, are $r = 20$ in., $L = 120$ in., and in both the inner and outer layers $h = \frac{1}{4}$ in., $E_1/(1 - \nu_1\nu_2) = 6 \times 10^6$ psi, $E_2/(1 - \nu_1\nu_2) = 20 \times 10^6$ psi, $\nu_2 = \frac{1}{3}$, $E_{12} = 15 \times 10^5$ psi, $E_0 = 10^6$ psi, $h_0 = \frac{1}{2}$ in., $a = 120$ in., and $\sigma_0 = 240$ psi. The cylinder is clamped at one end (at $s = 0$, $U_\xi = U_\theta = W = \Phi_\xi = 0$) and unsupported at the other end (at $s = \bar{s}$, $N_\xi = \hat{N}_{\xi\theta} = \hat{Q}_\xi = M_\xi = 0$).

Coupling is introduced by making one of the integrands of the coefficients nonsymmetric while the other coefficients in the stress resultant-strain relations are held fixed. For example, from Eq. (3) of Ref. 1, $C_2^{(\mu)} = (1/E_0h_0^{\mu+1})\int[E_2/(1 - \nu_1\nu_2)]z^\mu dz$ and hence for the present case $C_2^{(0)} = 20$, $C_2^{(1)} = 0$, and $C_2^{(2)} = \frac{5}{3}$. However, if the properties are changed such that $E_2/(1 - \nu_1\nu_2)$ is 30×10^6 in the inner layer and 10×10^6 in the outer layer, then $C_2^{(0)} = 20$, $C_2^{(1)} = -\frac{5}{2}$, $C_2^{(2)} = \frac{5}{3}$ and hence, according to Eqs. (2) of Ref. 1, circumferential coupling has been included.

It should be noted that similar results can be obtained by varying $C_{12}^{(\mu)}$ and $G^{(\mu)}$ but that similar variations in $C_1^{(\mu)}$ will effect more than one coupling term since in that case the location of the reference surface will change [see Eq. (1), Ref. 1].

As a convenient measure of the degree of coupling the parameter δ is introduced: $\delta = (D_i - D_0)/(D_i + D_0)$, where D represents any one of the integrands in Eqs. (3) of Ref. 1 with $\mu = 0$, i the inner layer, and o the outer layer.

Specific calculations were performed for cylinders in which $E_2/(1 - \nu_1\nu_2)$ varied in the manner described previously over a range $-\frac{3}{4} \leq \delta \leq \frac{3}{4}$. The changes in radial displacement at the free end between the coupled and uncoupled cases are shown in Fig. 1 as a function of harmonic number n for various values of the coupling parameter δ. $W^{(n)}$ is the displacement at the free end for given values of n and δ, and $W_0^{(n)}$ is the displacement at the free end for the same n and $\delta = 0$. Since the calculations are meaningful only for integral values of n,

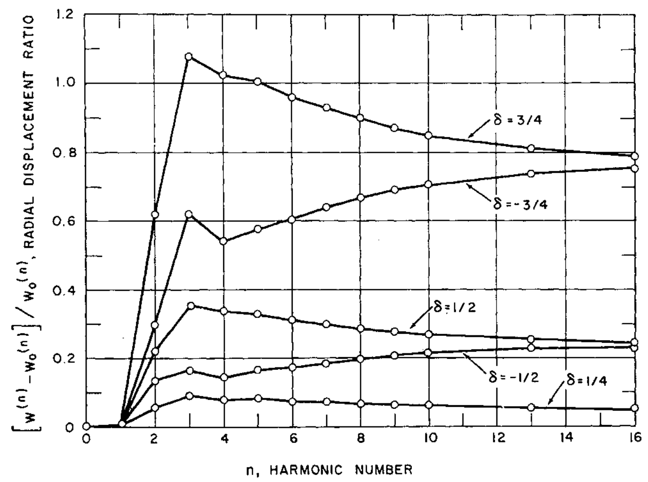

Fig. 1 Relative free-end deflections of an orthotropic cylinder with circumferential coupling.

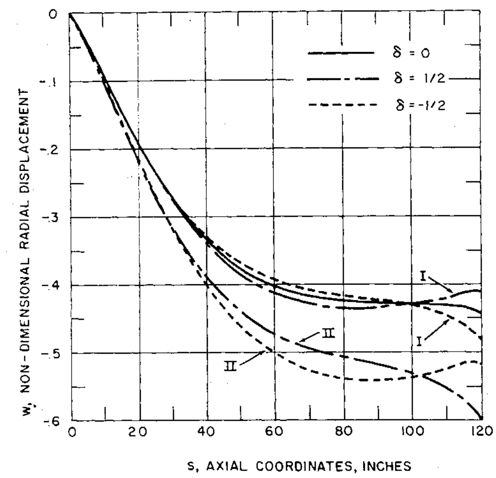

Fig. 2 Radial displacement profiles for a clamped-free orthotropic cylinder with $n = 3$ loading and I) Poisson coupling and II) circumferential coupling.

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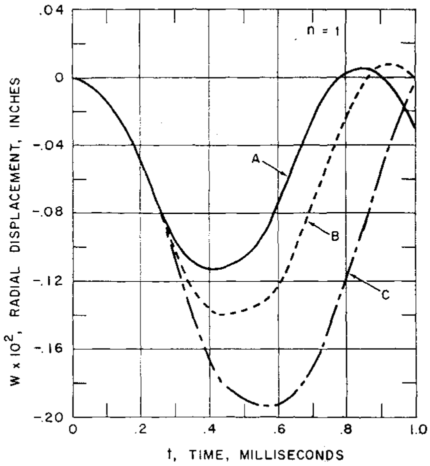

Fig. 3 Comparison of dynamic displacement histories for $n = 1$ at midspan of a cylinder with clamped boundaries. A) isotropic, B) ¾ isotropic shear modulus, C) ½ isotropic shear modulus.

the calculated points have been connected by straight lines to illustrate the trends.

It is of interest to note that, as harmonic number is increased past $n = 4$, the radial displacement ratios for $\delta > 0$ approach those for $\delta < 0$ and both appear to approach a horizontal tangent. This result is not immediately obvious from an examination of the coefficients of Eqs. (5) of Ref. 1. Similar calculations were performed for the same cylinder in which the values of E_1 and E_2 were interchanged and again E_2 was varied with essentially the same results. One pronounced difference in the cases studied was a maximum displacement difference occurring at $n = 4$ in the latter cases for $\delta > 0$ rather than at $n = 3$, as shown in Fig. 1. This result is not at all unlikely in practical situations involving composite layers in shells.

Nonzero values of $C_{12}^{(1)}$ and $G^{(1)}$ as well as variations in boundary conditions also produced similar effects. Nondimensional displacement profiles for the clamped-free cylinder subjected to a load $q = -\cos 3\theta$ are shown in Fig. 2. Coupling parameter values of $\delta = \pm\frac{1}{2}$ for both $C_{12}^{(1)}$ and $C_2^{(1)}$ variations, as well as the uncoupled case, $\delta = 0$, are presented. Note that increasing the circumferential stiffness in the inner layer while reducing it in the outer layer (i.e., $\delta = \frac{1}{2}$ for $C_2^{(1)}$) produces a displacement profile similar to the case where both the axial and circumferential Poisson's ratios are increased in the outer layer while being reduced in the inner layer (i.e., $\delta = -\frac{1}{2}$ for $C_{12}^{(1)}$). The major difference is that the latter produces displacements relatively close to the uncoupled results whereas the former produces displacements about 20–30% larger.

The similarity in the profiles for opposite signs in δ gives an indication of how Poisson's ratio is related to the direct stiffness. Increasing Poisson's ratio in one layer is, in some respects, equivalent to reducing the circumferential stiffness in the same layer. An additional effect involving Poisson's ratio is noted from an examination of an axisymmetric normal load on a free-free two-layer cylinder. If the Poisson's ratios are the same in both layers, the axial moment m_ξ will be identically zero over the length of the cylinder, whereas if coupling of the Poisson ratio terms is introduced, an axial moment will exist along the length of the cylinder with the exception of the free ends. This result is not restricted to orthotropic materials, but will also appear for isotropic layers since it is apparently a consequence of the Kirchoff hypothesis, which requires the cross section to remain plane. The Poisson coupling tends to distort the cross section and is restrained from doing so by the axial moment.

In another example it was of interest to evaluate the effects of "reduced" shear modulus in composite materials, which in most cases is considerably lower than the corresponding isotropic shear modulus $E_{12} = E/2\,(1 + \nu)$. A cylinder, clamped at both ends ($s = 0, \bar{s}$: $U_\xi = U_\theta = W = \Phi_\xi = 0$) and subjected to a normal triangular blast loading $q = -g(t)\cos n\theta$, is studied. Here,

$$g(t) = \begin{cases} (0.25 \times 10^6)t, & t \leq 4 \times 10^{-6} \\ (0.01 - t)/0.009996, & 4 \times 10^{-6} \leq t \leq 10^{-2} \\ 0, & t \geq 10^{-2} \end{cases}$$

The geometry and material properties are $r = 10$ in., $L = 60$ in., $h = \frac{1}{4}$ in., $\bar{\rho} = 0.2 \times 10^{-4}$ lb sec²/in.³, $E_1/(1 - \nu_1\nu_2) = E_2/(1 - \nu_1\nu_2) = 10^7$ psi, $\nu_1 = \nu_2 = 0.15$, and three values of the shear modulus: A) $E_{12} = 0.425 \times 10^7$ psi (isotropic), B) $E_{12} = 0.31875 \times 10^7$ psi, and C) $E_{12} = 0.2125 \times 10^7$ psi.

The normal displacement response for the $n = 1$ harmonic is shown for the midspan location in Fig. 3 and, as expected, the frequency is decreasing with shear modulus since this corresponds to an over-all reduction in stiffness of the structure. It is of interest to note that a 50% reduction in shear modulus (case C) produces almost twice the maximum deflection of the isotropic case A.

It is apparent that omission of the coupling coefficients [$\mu = 1$ in Eqs. (3), Ref. 1] can lead to important errors in the solution of multilayer shell problems. Although the numerical results were presented for cylindrical shells, it appears evident, from an investigation of the coefficients of Eqs. (5) of Ref. 1, that for general shells of revolution omission of the coupling terms can similarly lead to large errors. The use of isotropic theories for materials whose properties differ from isotropic by virtue of a "reduced" shear modulus can, as evidenced by the last example, also be erroneous. The present results are an initial effort to evaluate some effects of material coupling, and although the results cannot be stated in general terms, they indicate a need for further study.

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Response Characteristics of Thin Foil Heat Flux Sensors

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Introduction

THE asymptotic or Gardon type heat flux transducer has become increasingly important in the aerospace industry. Among the advantages of this type of transducer are direct readout and rapid response to time varying heat fluxes.

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